

# Looking for chiral anomaly in $K\gamma \rightarrow K\pi$ reactions

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## Abstract

In an experiment currently being performed at the Institute for High Energy Physics, Serpukhov, Russia, a beam of charged kaons is directed on a copper target. In the electromagnetic field of the target nuclei, two reactions occur:  $K^+\gamma \rightarrow K^+\pi^0$  and  $K^+\gamma \rightarrow K^0\pi^+$ . A peculiar distinction between these two reactions is that there is a chiral anomaly contribution in the former reaction, but not in the latter. This contribution can be directly seen through comparison of the cross sections of these reactions near the threshold. We derive expressions for these cross sections taking into account the anomaly and the contribution of the lightest vector mesons.

## 1 Introduction

Although the subject of the present paper is the analysis of  $K^+\gamma \rightarrow K\pi$  amplitudes near the threshold, let us begin by reminding an analogous consideration performed in the literature for the  $\pi^+\gamma \rightarrow \pi^+\pi^0$  amplitude.

$SU(2)_L \times SU(2)_R$  chiral symmetry holds due to the smallness of light quark masses:  $m_{u,d} \ll \Lambda_{\text{QCD}}$ . The anomaly in the divergence of the axial vector current allows us to obtain several amplitudes describing interactions of pions with photons at low energies, the most famous being the amplitude of a  $\pi^0 \rightarrow \gamma\gamma$  decay [1] (see also [2]).

An expression for the  $\pi\gamma \rightarrow \pi\pi$  amplitude is given by the chiral anomaly in the limit of small momenta of photon and pions [3]. The corresponding interaction Lagrangian is

$$\mathcal{L}_\pi = -\frac{ie}{8\pi^2 F_\pi^3} \varepsilon^{\mu\nu\rho\sigma} F_{\mu\nu} \pi^0 \partial_\rho \pi^+ \partial_\sigma \pi^+, \quad (1)$$

where  $F_\pi = 92.2$  MeV is the  $\pi \rightarrow \ell\nu$  decay constant [4, p. 1026].<sup>1</sup> The same expression was obtained almost simultaneously in other papers [5, 6].

In momentum representation,

$$A_\pi = -h \varepsilon^{\mu\nu\rho\sigma} q_\mu \epsilon_\nu p_\rho k_{1\sigma}, \quad (2)$$

where  $h = e/(4\pi^2 F_\pi^3)$ , and  $q$ ,  $p$ , and  $k_1$  are four-momenta of the photon, initial  $\pi^+$ , and final  $\pi^+$  respectively.

The way to check (1) and (2) by studying  $\pi^0$  production in a beam of charged pions which scatter coherently in the Coulomb field of heavy nucleus was suggested in [7]. To estimate the corrections to (2), it was assumed in [7] that variation of the function  $h$  near the threshold comes mainly from the vector meson exchange diagrams, and the following expression was obtained:<sup>2</sup>

$$M_\pi = A_\pi \left\{ 1 + \frac{2f_{\rho\pi\gamma}f_{\rho\pi\pi}}{m_\rho^2 h} \left[ \frac{s}{m_\rho^2 - s} + \frac{t}{m_\rho^2 - t} + \frac{u}{m_\rho^2 - u} \right] + \frac{ef_{\omega\gamma}f_{\omega 3\pi}}{m_\omega^2 h} \frac{q^2}{m_\omega^2 - q^2} \right\}, \quad (3)$$

where  $s = (p+q)^2$ ,  $t = (p-k_1)^2$ , and  $u = (q-k_1)^2$  are the Mandelstam variables for the reaction  $\pi^+\gamma^* \rightarrow \pi^+\pi^0$ ;  $s+t+u = 3m_\pi^2 + q^2$ . Subtraction is made in (3) since, in the limit  $s, t, u, q^2 \rightarrow 0$ , only the anomaly contribution should survive. The equivalent photon approximation was used in [7] in order to obtain the cross section of the reaction  $\pi^+ \rightarrow \pi^+\pi^0$  in the Coulomb field of the nucleus from the cross section of the reaction  $\pi^+\gamma \rightarrow \pi^+\pi^0$ .

Experimental verification of formulas (2) and (3) is described in [9]. In the experiment, a 40 GeV pion beam from the IHEP proton accelerator was used to produce neutral pions in the Coulomb fields of C, Al and Fe nuclei. According to [9],

$$F_{3\pi}(0) \equiv h = 12.9 \pm 0.9 \pm 0.5 \pm 1.0 \text{ GeV}^{-3}, \quad (4)$$

<sup>1</sup>In [4],  $f_{\pi^-} = F_\pi\sqrt{2}$  is used.

<sup>2</sup>For  $\sqrt{s} \gtrsim 4m_\pi$ , the  $\rho$ -meson width should be taken into account in its  $s$  channel contribution; see Fig. 4 and Eq. (14) in [8].

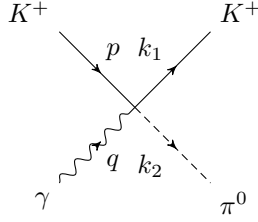


Figure 1: Momenta of particles in the  $K^+\gamma \rightarrow K^+\pi^0$  reaction.

while the theoretical number is

$$h = \frac{e}{4\pi^2 F_\pi^3} = 9.8 \text{ GeV}^{-3}. \quad (5)$$

In expression (4), the first error is statistical, the second error is systematic, and the third error comes from the unknown phase of the  $\rho$  exchange contribution in (3) (the  $\omega$  contribution is negligible).

Thus, in the case of pions, the anomaly saturates the considered amplitude in the kinematics of the experiment [9].<sup>3</sup>

With the inclusion of the chiral one- and two-loop corrections [10, 11] and electromagnetic corrections [12], a reanalysis of the Serpukhov data leads to the value  $F_{3\pi} = 10.7 \pm 1.2 \text{ GeV}^{-3}$ ; see also [13]. The COMPASS Collaboration had plans to measure  $F_{3\pi}$  with better accuracy [14, Sec. 4.2].

As long as the strange quark mass can be considered small in comparison with  $\Lambda_{\text{QCD}}$ , chiral symmetry is generalized to  $SU(3)_L \times SU(3)_R$  and amplitudes containing  $K$  mesons can be predicted from a consideration of the anomaly as well. This was done in [5], where, in particular, the anomaly contribution to the  $K^+\gamma \rightarrow K^+\pi^0$  amplitude was found. It appears to be similar to (1) and (2):<sup>4</sup>

$$\mathcal{L}_K = -\frac{ie}{8\pi^2 F_\pi^3} \varepsilon^{\mu\nu\rho\sigma} F_{\mu\nu} \pi^0 \partial_\rho K^+ \partial_\sigma \overline{K}^+, \quad (6)$$

$$A = -\frac{e}{4\pi^2 F_\pi^3} \varepsilon^{\mu\nu\rho\sigma} q_\mu \epsilon_\nu p_\rho k_{1\sigma}, \quad (7)$$

where the particles' momenta are defined as in Fig. 1. Analogously to the case of pions, the manifestation of anomaly (7) can be looked for in  $\pi^0$  production in the beam of charged kaons scattered coherently in the Coulomb field of heavy nucleus. Such an experiment is under way at Serpukhov [15], where scattering of a  $K^+$  beam with energy  $E_K = 18 \text{ GeV}$  in the Coulomb field of a copper (Cu) nucleus with coherent  $K\pi$  production is studied.

There are two aspects in which the case of kaons differs from the case of pions. On the one hand, since kaons are relatively heavy, we cannot approach the point  $s = t = u = q^2 = 0$  in which the anomaly dominates as closely as in the case of the pions. On the other hand, in the case of charged kaons, there are two reaction channels,  $K^+\gamma^* \rightarrow K^+\pi^0$  and  $K^+\gamma^* \rightarrow K^0\pi^+$ , and only the first is influenced by the chiral anomaly [5]. Thus, comparing experimental data on  $K^+\pi^0$  and  $K^0\pi^+$  production close to the threshold, one can hope to observe the effect of the anomaly.

To clarify why the amplitude of the reaction  $K^+\gamma \rightarrow K^+\pi^0$  contains the anomaly while that of  $K^+\gamma \rightarrow K^0\pi^+$  is anomaly free, one should look at Fig. 2, where, for completeness, diagrams for the reaction  $\pi^+\gamma \rightarrow \pi^+\pi^0$  are shown as well. The photon should be attached to Pauli-Villars fields running inside the triangle diagrams drawn in Fig. 2 in all possible ways, leading to proportionality of the sum of the corresponding box diagrams to the sum of electric charges of regulators. It gives  $\frac{2}{3} + \frac{2}{3} - \frac{1}{3} = 1$  for Fig. 2a,  $\frac{2}{3} - \frac{1}{3} - \frac{1}{3} = 0$  for Fig. 2b,  $\frac{2}{3} - \frac{1}{3} - \frac{1}{3} = 0$  for Fig. 2c and, finally,  $\frac{2}{3} + \frac{2}{3} - \frac{1}{3} = 1$  for Fig. 2d. Thus, the anomaly contributions to the processes  $\pi^+\gamma \rightarrow \pi^+\pi^0$  and  $K^+\gamma \rightarrow K^+\pi^0$  are equal, while that to the process  $K^+\gamma \rightarrow K^0\pi^+$  equals zero. For an appropriate presentation of the calculation of the anomalous amplitudes with the help of regulator fields, see textbook [16, Ch. 6a].

We have the following plan for this paper. In Sec. 2, formulas for the cross sections of the reactions  $K^+\gamma \rightarrow K^+\pi^0$  and  $K^+\gamma \rightarrow K^0\pi^+$  at low  $s$  are obtained. In Sec. 3, with the help of the equivalent photon approximation, these formulas are converted into the expressions for the cross sections of the  $K^+N \rightarrow K^+\pi^0N$  and  $K^+N \rightarrow K^0\pi^+N$  reactions. Comparing corresponding plots, we will conclude that extraction of the anomaly contribution into the first reaction from the experimental data should be possible. We summarize our results in the Conclusions. In Appendix A we present the “back of the envelope” derivation of the induced by the anomaly part of the amplitude cross section of the reaction  $K^+\gamma \rightarrow K^+\pi^0$ , while in Appendix B numerical values of the coupling constants used in Sec. 2 are derived.

<sup>3</sup> In [9], events with  $s < 10m_\pi^2$ ,  $|t| < 3.5m_\pi^2$  were selected. Photon virtuality varies in the following interval:  $2 \cdot 10^{-3} \text{ GeV}^2 > -q^2 > ((s - m_\pi^2)/2E_\pi)^2$ ,  $E_\pi = 40 \text{ GeV}$ .

<sup>4</sup> The coefficients in (6) and (7) are three times bigger than in [5] due to an extra color factor,  $N_c = 3$ .

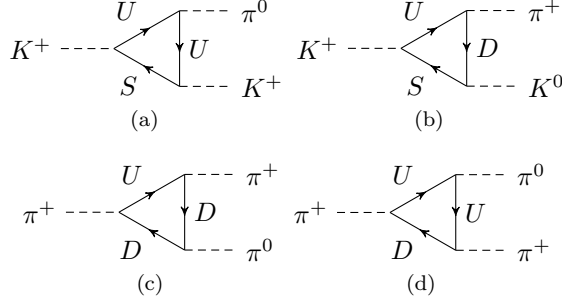


Figure 2: Attaching photon to the Pauli-Villars regulator fields, we get box diagrams describing anomaly contributions to (a) the  $K^+\gamma \rightarrow K^+\pi^0$ , (b)  $K^+\gamma \rightarrow K^0\pi^+$  and (c), (d)  $\pi^+\gamma \rightarrow \pi^+\pi^0$  reactions.

## 2 Cross sections of $K^+\gamma \rightarrow K^+\pi^0$ and $K^+\gamma \rightarrow K^0\pi^+$ reactions at low invariant masses

Let us start with the calculation of the anomaly contribution to the cross section. Momenta of particles are shown in Fig. 1,  $s = (p+q)^2$ ,  $t = (p-k_1)^2$ , and  $u = (p-k_2)^2$  are Mandelstam variables, and we neglect photon virtuality in this section,  $q^2 = 0$ . The standard formula for a differential cross section is

$$\frac{d\sigma_r}{dt} = \frac{\overline{|A|^2}}{16\pi(s-m_{K^+}^2)^2}, \quad (8)$$

where  $\overline{|A|^2}$  stands for the square of the amplitude averaged over the two transversal photon polarizations. One should use the expression for  $A$  from (7), obtaining

$$\overline{|A|^2} = \frac{e^2}{128\pi^4 F_\pi^6} [-t(s-m_{K^+}^2)^2 - t^2 s + m_{\pi^0}^2 t(s+m_{K^+}^2) - m_{K^+}^2 m_{\pi^0}^4], \quad (9)$$

and integrate (8) in the following interval:

$$\begin{aligned} \frac{m_{\pi^0}^4}{4s} - \left( \frac{s-m_{K^+}^2}{2\sqrt{s}} + \frac{\sqrt{[s-(m_{K^+}+m_{\pi^0})^2][s-(m_{K^+}-m_{\pi^0})^2]}}{2\sqrt{s}} \right)^2 < t \\ < \frac{m_{\pi^0}^4}{4s} - \left( \frac{s-m_{K^+}^2}{2\sqrt{s}} - \frac{\sqrt{[s-(m_{K^+}+m_{\pi^0})^2][s-(m_{K^+}-m_{\pi^0})^2]}}{2\sqrt{s}} \right)^2, \end{aligned} \quad (10)$$

getting the following result:

$$\sigma_r = \frac{\alpha}{3 \cdot 2^{10} \pi^4 F_\pi^6} \frac{s-m_{K^+}^2}{s^2} \{ [s-(m_{K^+}+m_{\pi^0})^2][s-(m_{K^+}-m_{\pi^0})^2] \}^{3/2}. \quad (11)$$

The simplicity of the final expression clearly demonstrates that a simple derivation of it should exist. Such a derivation is presented in Appendix A.

At low energies, the variation of the amplitude of the  $K^+\gamma \rightarrow K^+\pi^0$  reaction comes mainly from vector meson exchange diagrams presented in Fig. 3. The exchanged mesons are  $K^{*+}$  in the  $s$  and  $u$  channels, and  $\rho^0$ ,  $\omega$  and  $\phi$  mesons in the  $t$  channel. Similarly, for the  $K^+\gamma \rightarrow K^0\pi^+$  reaction important meson exchanges are  $K^{*+}$  in the  $s$  channel,  $K^{*0}$  in the  $u$  channel, and  $\rho^+$  in the  $t$  channel. The corresponding diagrams are presented in Fig. 4.

The amplitude for the diagram shown in Fig. 3a is

$$A_s^{(0)}(K^+\gamma \rightarrow K^+\pi^0) = \frac{2f_{K^{*+}K^+\gamma}f_{K^{*+}K^+\pi^0}}{s-m_{K^{*+}}^2 + i\sqrt{s}\Gamma_{K^{*+}}(s)} \varepsilon^{\mu\nu\rho\sigma} q_\mu \epsilon_\nu p_\rho k_{1\sigma}, \quad (12)$$

where  $f_{K^{*+}K^+\gamma}$  and  $f_{K^{*+}K^+\pi^0}$  are coupling constants,

$$\Gamma_{K^{*+}}(s) = \Gamma_{K^{*+}} \frac{\sqrt{s}}{m_{K^{*+}}} \left[ \frac{\left(1 - \frac{(m_K - m_\pi)^2}{s}\right) \left(1 - \frac{(m_K + m_\pi)^2}{s}\right)}{\left(1 - \frac{(m_K - m_\pi)^2}{m_{K^{*+}}^2}\right) \left(1 - \frac{(m_K + m_\pi)^2}{m_{K^{*+}}^2}\right)} \right]^{\frac{3}{2}}, \quad (13)$$

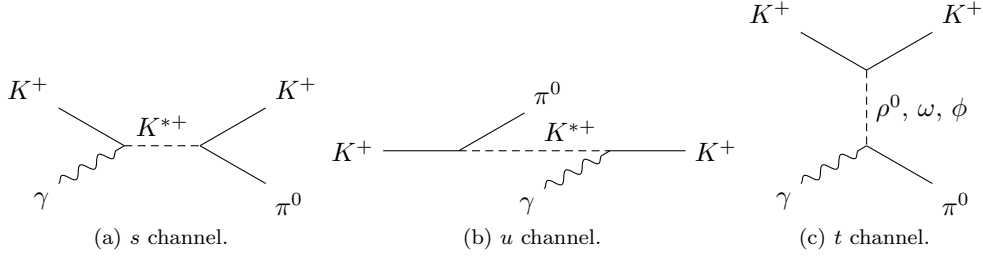


Figure 3: Vector meson exchanges in the process  $K^+\gamma \rightarrow K^+\pi^0$ :  $K^{*+}$  meson in (a)  $s$  and (b)  $u$  channels, and (c)  $\rho^0$ ,  $\omega$ ,  $\phi$  mesons in the  $t$  channel.

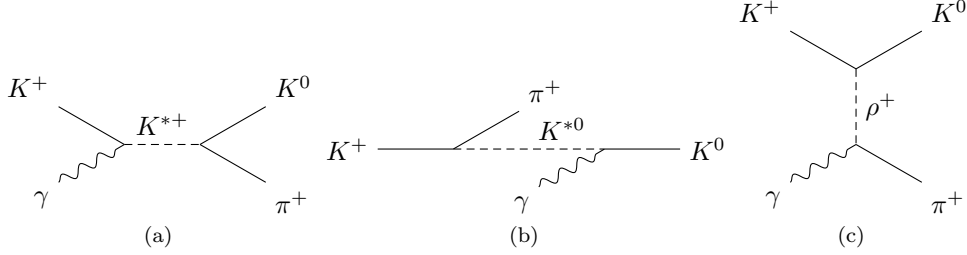


Figure 4: Vector meson exchanges in the process  $K^+\gamma \rightarrow K^0\pi^+$ : (a)  $K^{*+}$  in the  $s$  channel, (b)  $K^{*0}$  in the  $u$  channel, (c)  $\rho^+$  in the  $t$  channel.

and  $\Gamma_{K^{*+}}$  is the total width of  $K^{*+}$  as provided in [4]. Since at  $s, t, u \rightarrow 0$  only the anomaly contribution should survive, we subtract from  $A_s^{(0)}(K^+\gamma \rightarrow K^+\pi^0)$  its value at  $s = 0$ :

$$\begin{aligned} A_s(K^+\gamma \rightarrow K^+\pi^0) &= A_s^{(0)}(K^+\gamma \rightarrow K^+\pi^0) - A_s^{(0)}(K^+\gamma \rightarrow K^+\pi^0)|_{s=0} \\ &= -\frac{2f_{K^{*+}K^+\gamma}f_{K^{*+}K^+\pi^0}}{m_{K^{*+}}^2 - s - i\sqrt{s}\Gamma_{K^{*+}}(s)} \cdot \frac{s + i\sqrt{s}\Gamma_{K^{*+}}(s)}{m_{K^{*+}}^2} \varepsilon^{\mu\nu\rho\sigma} q_\mu \epsilon_\nu p_\rho k_{1\sigma}. \end{aligned} \quad (14)$$

Since  $\sqrt{s}\Gamma_{K^{*+}}(s) \ll s$ , we will neglect it in the numerator of (14). Performing similar calculations in the  $u$  and  $t$  channels [Figs. 3b and 3c], we notice that the physical regions of  $t < 0$  and  $u < m_{K^+}^2$  lie well below the thresholds for decay processes of the  $K^*$ ,  $\rho$ ,  $\omega$  and  $\phi$  mesons, so no imaginary terms appear in expressions for their amplitudes.

Using the same approach for diagrams in Fig. 4, we obtain the following expressions for the differential cross sections:

$$\begin{aligned} \frac{d\sigma(K^+\gamma \rightarrow K^+\pi^0)}{dt} &= \frac{1}{2^7\pi} \left( t + \frac{(st - m_{K^+}^2 m_{\pi^0}^2)(t - m_{\pi^0}^2)}{(s - m_{K^+}^2)^2} \right) \\ &\times \left| \frac{e}{4\pi^2 F_\pi^3} + \frac{2f_{K^{*+}K^+\gamma}f_{K^{*+}K^+\pi^0}}{m_{K^{*+}}^2 - s - i\sqrt{s}\Gamma_{K^{*+}}(s)} \cdot \frac{s}{m_{K^{*+}}^2} + \frac{2f_{K^{*+}K^+\gamma}f_{K^{*+}K^+\pi^0}}{m_{K^{*+}}^2 - u} \cdot \frac{u}{m_{K^{*+}}^2} \right. \\ &\quad \left. + \frac{2f_{\rho^0\pi^0\gamma}f_{\rho^0 K^+K^+}}{m_{\rho^0}^2 - t} \cdot \frac{t}{m_{\rho^0}^2} + \frac{2f_{\omega\pi^0\gamma}f_{\omega K^+K^+}}{m_\omega^2 - t} \cdot \frac{t}{m_\omega^2} + \frac{2f_{\phi\pi^0\gamma}f_{\phi K^+K^+}}{m_\phi^2 - t} \cdot \frac{t}{m_\phi^2} \right|^2, \end{aligned} \quad (15)$$

$$\begin{aligned} \frac{d\sigma(K^+\gamma \rightarrow K^0\pi^+)}{dt} &= -\frac{stu - sm_{K^0}^2 m_{\pi^+}^2 - tm_{K^+}^2 m_{K^0}^2 - um_{K^+}^2 m_{\pi^+}^2 + 2m_{K^+}^2 m_{K^0}^2 m_{\pi^+}^2}{2^7\pi(s - m_{K^+}^2)^2} \\ &\times \left| \frac{2f_{K^{*+}K^+\gamma}f_{K^{*+}K^0\pi^+}}{m_{K^{*+}}^2 - s - i\sqrt{s}\Gamma_{K^{*+}}(s)} \cdot \frac{s}{m_{K^{*+}}^2} + \frac{2f_{K^{*0}K^0\gamma}f_{K^{*0}K^+\pi^+}}{m_{K^{*0}}^2 - u} \cdot \frac{u}{m_{K^{*0}}^2} \right. \\ &\quad \left. - \frac{2f_{\rho^+\pi^+\gamma}f_{\rho^+ K^+K^0}}{m_{\rho^+}^2 - t} \cdot \frac{t}{m_{\rho^+}^2} \right|^2, \end{aligned} \quad (16)$$

Coupling constants  $f_i$  are defined in Appendix B with numerical values presented in Table 1. In (15), the most important correction comes from the  $K^*$  exchange in the  $s$  channel. The contribution of the  $\phi$ -meson exchange in the  $t$  channel is approximately 20 times smaller than the sum of the  $\rho$ - and  $\omega$ -mesons contributions, so it could be safely neglected. In (16), the term with  $K^{*+}$  exchange dominates.

Table 1: Coupling constants required to calculate cross sections of the  $K^+\gamma \rightarrow K^+\pi^0$  and  $K^+\gamma \rightarrow K^0\pi^+$  reactions. Let us stress that the signs of all of the  $f_{VPP}$  constants could be changed simultaneously, as can all the signs of  $f_{VP\gamma}$ , leading to two solid curves in Fig. 5.

$f_{K^*+K^+\pi^0}$	=	3.10
$f_{K^*+K^0\pi^+}$	=	4.38
$f_{K^{*0}K^+\pi^+}$	=	4.41
$f_{\rho^0 K^+K^+}$	=	3.16
$f_{\rho^+ K^+K^0}$	=	-4.47
$f_{\omega K^+K^+}$	=	3.16
$f_{\phi K^+K^+}$	=	-4.47
$f_{K^*+K^+\gamma}$	=	0.240 GeV <sup>-1</sup>
$f_{K^{*0}K^0\gamma}$	=	-0.385 GeV <sup>-1</sup>
$f_{\rho^0\pi^0\gamma}$	=	0.252 GeV <sup>-1</sup>
$f_{\rho^+\pi^+\gamma}$	=	0.219 GeV <sup>-1</sup>
$f_{\omega\pi^0\gamma}$	=	0.696 GeV <sup>-1</sup>
$ f_{\phi\pi^0\gamma} $	=	0.040 GeV <sup>-1</sup>

### 3 Cross sections of $K^+N \rightarrow K^+\pi^0N$ and $K^+N \rightarrow K^0\pi^+N$ reactions in the equivalent photon approximation

In the equivalent photon approximation, the following formula for the cross section of pion production in the Coulomb field of a nucleus  $N$  holds [17]:

$$\frac{d\sigma(K^+N \rightarrow K\pi N)}{dtds dq_\perp^2} = \frac{Z^2\alpha}{\pi} \frac{q_\perp^2}{[q_\perp^2 + (s - m_{K^+}^2)^2/(4E_K^2)]^2(s - m_{K^+}^2)} \frac{d\sigma(K^+\gamma \rightarrow K\pi)}{dt} \cdot |F(\vec{q}^2)|^2, \quad (17)$$

where the nucleus form factor  $F(\vec{q}^2)$  is taken into account,

$$F(\vec{q}^2) = \exp\left(-\frac{\langle r^2 \rangle \vec{q}^2}{6}\right), \quad (18)$$

$\langle r^2 \rangle$  is the mean-square radius of the nucleus,  $\langle r^2 \rangle^{1/2} = r_0 A^{1/3}$ ,  $r_0 = 0.94$  fm, and  $A$  is the number of nucleons in the nucleus,  $A = 63$  for copper. In (18),  $\vec{q}^2 \equiv \vec{q}_\perp^2 + \vec{q}_\parallel^2$ , but, since for  $s = 0.5$  GeV<sup>2</sup> we have

$$a \equiv \frac{\langle r^2 \rangle \vec{q}_\parallel^2}{3} = \frac{\langle r^2 \rangle}{3} \left( \frac{s - m_{K^+}^2}{2E_K} \right)^2 = 6 \cdot 10^{-3} \ll 1,$$

we can safely neglect  $\vec{q}_\parallel^2$  there. Integration over  $\vec{q}_\perp^2$  from zero to infinity results in

$$\frac{d\sigma(K^+N \rightarrow K\pi N)}{dtds} = \frac{Z^2\alpha}{\pi} \cdot \frac{E_1(a) - 1}{s - m_{K^+}^2} \cdot \frac{d\sigma(K^+\gamma \rightarrow K\pi)}{dt}, \quad (19)$$

where  $E_1(a)$  is the exponential integral,

$$E_1(a) = \int_a^\infty \frac{e^{-z}}{z} dz. \quad (20)$$

In order to obtain differential cross sections  $d\sigma/ds$  of the reactions under study as functions of the invariant mass of the produced  $K\pi$  system, Eq. (19) is numerically integrated over  $t$ . The result of the integration is presented in Fig. 5. The effect of the anomaly can be seen through comparison of  $K^+\pi^0$  and  $K^0\pi^+$  productions at  $s \lesssim 0.55$  GeV<sup>2</sup>, where the anomaly contribution is the largest.

In Fig. 6, the cross section of the reaction  $K^+N \rightarrow K^+\pi^0N$  is presented and compared to that without the anomaly contribution.

### 4 Conclusions

The main results of this paper are given by formulas (15) and (16) and are shown in Fig. 5, where two solid lines for the cross section of  $K^+N \rightarrow K^+\pi^0N$  reaction correspond to positive and negative signs of the product of the

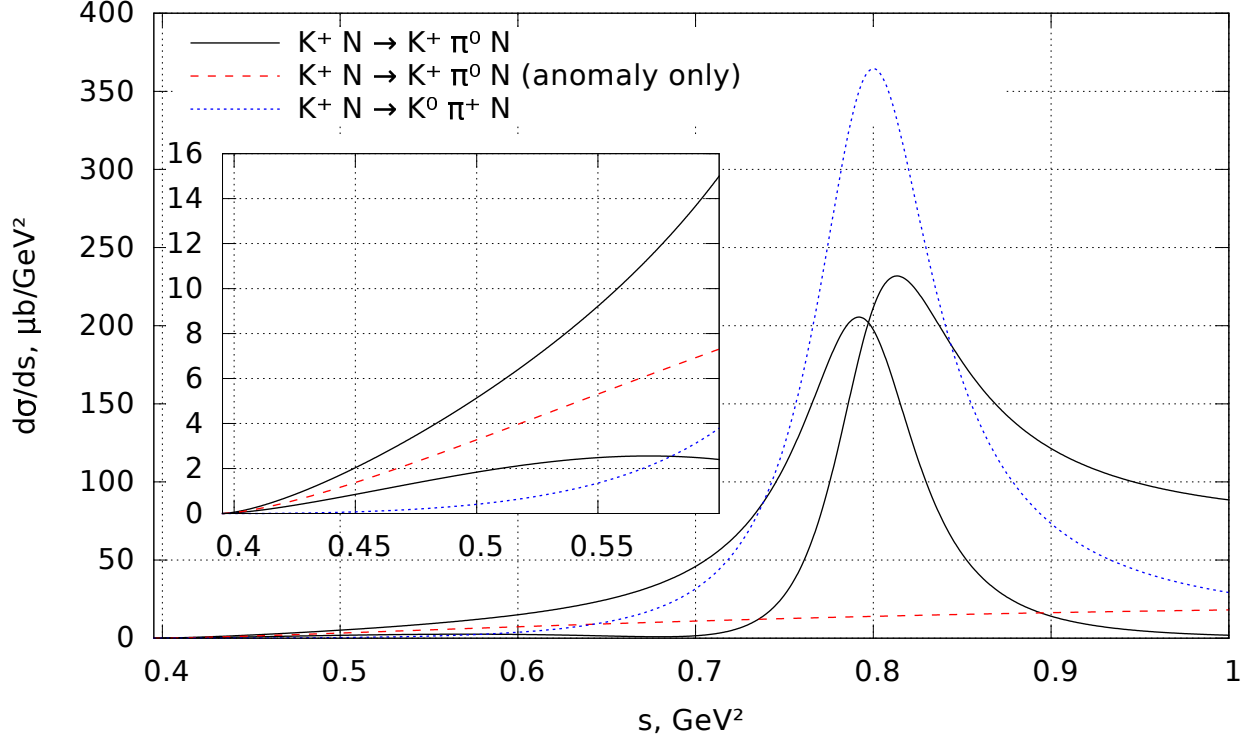


Figure 5: Differential cross sections of the reactions  $K^+N \rightarrow K^+\pi^0 N$  and  $K^+N \rightarrow K^0\pi^+ N$  for  $N = {}^{63}\text{Cu}$ . The two solid lines for the  $K^+N \rightarrow K^+\pi^0 N$  reaction correspond to the two possible choices of the signs of the product of coupling constants (see Appendix B).

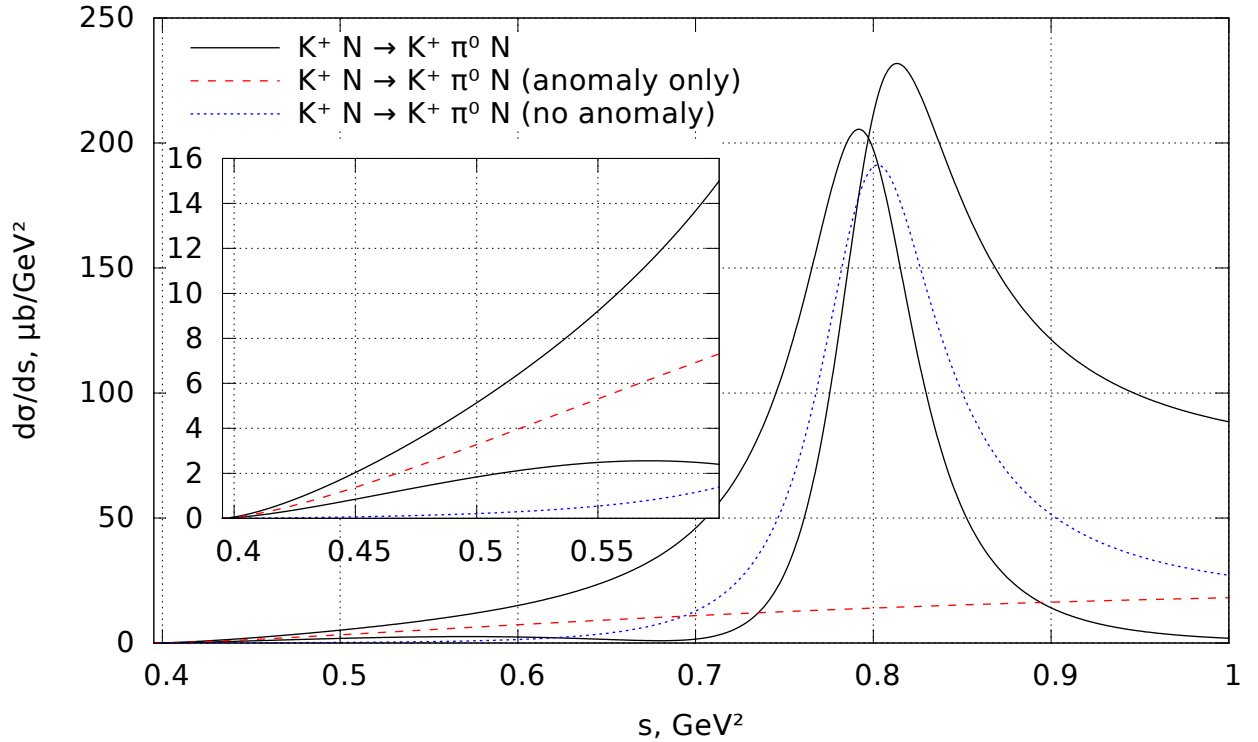


Figure 6: Differential cross sections of the reaction  $K^+N \rightarrow K^+\pi^0 N$  for  $N = {}^{63}\text{Cu}$ . The two solid lines correspond to the two possible choices of the signs of the product of coupling constants.

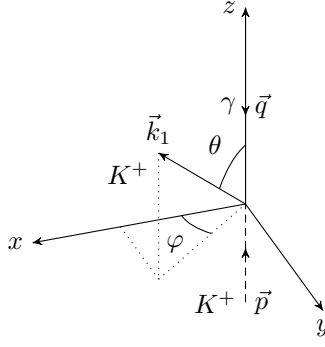


Figure 7: Kinematics of the  $K^+\gamma \rightarrow K^+\pi^0$  reaction in the center of mass system. Initial particles are moving along the  $z$  axis.

coupling constants  $f_i$ . The luminosity of  $60 \mu\text{b}^{-1}$  is planned to be collected in the experiment currently being performed at Serpukhov (with the account for the detector efficiency near the  $K\pi$  threshold) [18]. Integrating subplots of Fig. 5, we get that either 20 or 70  $K^+\pi^0$  production events will be observed in the interval  $0.4 < s < 0.6 \text{ GeV}^2$  for the destructive or constructive interference of the anomaly and resonances terms. As for the  $K^0\pi^+$  production, about 10 events should be observed in that  $s$  interval. Thus, one can hope to observe a manifestation of the chiral anomaly in future Serpukhov data.

Let us compare our results with those of [19, 20], where the photoproduction of pions in a charged kaon beam was studied under conditions of the IHEP experiment as well. The main difference is that in our paper the amplitudes describing vector meson ( $K^*$ ,  $\rho$ ,  $\omega$ ,  $\phi$ ) contributions are subtracted at zero momenta. This subtraction is needed since only the anomaly contribution remains at zero momenta. In [20], only the  $K^+\gamma \rightarrow K^+\pi^0$  reaction is considered. Apart from the subtraction, our Eq. (15) differs from Eq. (4) in [20] by an extra factor  $-2$  in  $u$  and  $t$  channel contributions. Also, only the enhancement of the anomaly contribution by the interference with that of the intermediate vector bosons is presented in the figures in [20]. In [19], expressions for the cross sections have the wrong dimension.

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## A Simple derivation of the cross section induced by the anomaly

Let us consider the reaction  $K^+\gamma \rightarrow K^+\pi^0$  in the center of mass system; see Fig. 7. According to (7), the expression for the amplitude looks like

$$A = \frac{e}{4\pi^2 F_\pi^3} \varepsilon^{\mu\nu\rho\sigma} \epsilon_\mu q_\nu p_\rho k_{1\sigma}, \quad (\text{A1})$$

where  $\epsilon_\mu$  is the polarization vector of the photon. Since both  $p_\rho$  and  $q_\nu$  have only their third ( $z$ ) spatial components differing from zero, one of them should contribute by its temporal ( $t$ ) component. As a result, we get

$$A = \frac{e}{4\pi^2 F_\pi^3} \varepsilon^{\mu 0 3 \sigma} \epsilon_\mu k_{1\sigma} (E_K E_\gamma + E_\gamma^2), \quad (\text{A2})$$

where  $E_K$  and  $E_\gamma$  are the energies of the incoming  $K^+$  and the photon. When the photon polarization is parallel to the  $x$  axis ( $\epsilon_x = \epsilon_1$ ), the  $y$  component of  $\vec{k}_1$  contributes

$$A_1 = \frac{e}{4\pi^2 F_\pi^3} |\vec{k}_1| \sin \theta \sin \varphi (E_K E_\gamma + E_\gamma^2). \quad (\text{A3})$$

When the photon polarization is parallel to the  $y$  axis ( $\epsilon_y = \epsilon_2$ ), the  $x$  component of  $\vec{k}_1$  contributes

$$A_2 = \frac{e}{4\pi^2 F_\pi^3} |\vec{k}_1| \sin \theta \cos \varphi (E_K E_\gamma + E_\gamma^2). \quad (\text{A4})$$

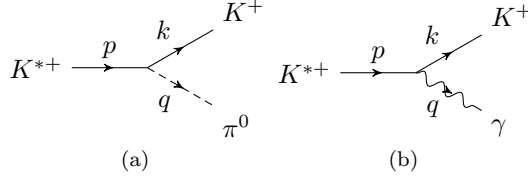


Figure 8: Diagrams used for calculation of vector mesons coupling constants: (a)  $f_{K^{*+}K^+\pi^0}$ , (b)  $f_{K^{*+}K^+\gamma}$ .

From the general formula for the differential cross section of the  $2 \rightarrow 2$  reaction in the center of mass system, averaging squares of obtained amplitudes over photon polarizations, we get

$$d\sigma = \frac{|A(K^+\gamma \rightarrow K^+\pi^0)|^2}{64\pi^2} \frac{|\vec{p}'|}{|\vec{p}|\varepsilon^2} d\phi = \frac{\alpha}{28\pi^5 F_\pi^6} |\vec{k}_1|^2 \cdot \frac{1}{2} \sin^2 \theta E_\gamma^2 (E_K + E_\gamma)^2 \frac{|\vec{k}_1|}{E_\gamma (E_K + E_\gamma)^2} d\phi d\cos\theta. \quad (\text{A5})$$

Integrating the differential cross section over angles and taking into account that in the center of mass system  $E_\gamma = \frac{s - m_{K^+}^2}{2\sqrt{s}}$ ,  $|\vec{k}_1| = \frac{\{[s - (m_{K^+} + m_{\pi^0})^2][s - (m_{K^+} - m_{\pi^0})^2]\}^{1/2}}{2\sqrt{s}}$ , we get

$$\sigma_r = \frac{\alpha}{3 \cdot 2^{10} \pi^4 F_\pi^6} \frac{s - m_{K^+}^2}{s^2} \{[s - (m_{K^+} + m_{\pi^0})^2][s - (m_{K^+} - m_{\pi^0})^2]\}^{3/2}, \quad (\text{A6})$$

which coincides with (11).

## B Coupling constants of intermediate vector bosons

In order to calculate cross sections through Eqs. (15) and (16), we need to know the numerical values of the coupling constants  $f_i$ . These constants are defined through the following interaction lagrangian:

$$\begin{aligned} \mathcal{L}_I = & f_{K^{*+}K^+\gamma} \varepsilon^{\mu\nu\alpha\beta} \partial_\mu A_\nu \partial_\alpha K_\beta^{*+} \overline{K^+} + i f_{K^{*+}K^+\pi^0} K_\mu^{*+} (\overline{K^+} \partial^\mu \pi^0 - \pi^0 \partial^\mu \overline{K^+}) \\ & + i f_{K^{*+}K^0\pi^+} K_\mu^{*+} (\overline{K^0} \partial^\mu \pi^+ - \pi^+ \partial^\mu \overline{K^0}) \\ & + f_{K^{*0}K^0\gamma} \varepsilon^{\mu\nu\alpha\beta} \partial_\mu A_\nu \partial_\alpha K_\beta^{*0} \overline{K^0} + i f_{K^{*0}K^0\pi^+} K_\mu^{*0} (\overline{K^+} \partial^\mu \pi^+ - \pi^+ \partial^\mu \overline{K^+}) \\ & + f_{\rho^+\pi^+\gamma} \varepsilon^{\mu\nu\alpha\beta} \partial_\mu A_\nu \partial_\alpha \rho_\beta^+ \overline{\pi^+} + i f_{\rho^+K^+K^0} \rho_\mu^+ (\overline{K^+} \partial^\mu K^0 - K^0 \partial^\mu \overline{K^+}) \\ & + \frac{1}{2} f_{\rho^0\pi^0\gamma} \varepsilon^{\mu\nu\alpha\beta} \partial_\mu A_\nu \partial_\alpha \rho_\beta^0 \pi^0 + \frac{i}{2} f_{\rho^0K^+K^+} \rho_\mu^0 (K^+ \partial^\mu \overline{K^+} - \overline{K^+} \partial^\mu K^+) \\ & + \frac{1}{2} f_{\omega\pi^0\gamma} \varepsilon^{\mu\nu\alpha\beta} \partial_\mu A_\nu \partial_\alpha \omega_\beta \pi^0 + \frac{i}{2} f_{\omega K^+K^+} \omega_\mu (K^+ \partial^\mu \overline{K^+} - \overline{K^+} \partial^\mu K^+) \\ & + \frac{1}{2} f_{\phi\pi^0\gamma} \varepsilon^{\mu\nu\alpha\beta} \partial_\mu A_\nu \partial_\alpha \phi_\beta \pi^0 + \frac{i}{2} f_{\phi K^+K^+} \phi_\mu (K^+ \partial^\mu \overline{K^+} - \overline{K^+} \partial^\mu K^+) \\ & + \text{h.c.} \end{aligned} \quad (\text{B1})$$

Absolute values of some of the constants  $f_i$  are obtained from the partial widths. Other constants are deduced from these values with the help of the  $SU(3)$  symmetry between the  $u$ ,  $d$ , and  $s$  quarks, which also determines the relative signs of the coupling constants. Results of the calculations presented below are summarized in Table 1.

First, consider the  $K^{*+} \rightarrow K^+\pi^0$  decay. Its amplitude equals (see Fig. 8a where momenta of particles are shown)

$$A(K^{*+} \rightarrow K^+\pi^0) = f_{K^{*+}K^+\pi^0} K_\mu^{*+} (k - q)^\mu K^+\pi^0. \quad (\text{B2})$$

Its width

$$\Gamma(K^{*+} \rightarrow K^+\pi^0) = \frac{f_{K^{*+}K^+\pi^0}^2 m_{K^{*+}}}{48\pi} \left\{ \left[ 1 - \left( \frac{m_{K^+} + m_{\pi^0}}{m_{K^{*+}}} \right)^2 \right] \left[ 1 - \left( \frac{m_{K^+} - m_{\pi^0}}{m_{K^{*+}}} \right)^2 \right] \right\}^{3/2}. \quad (\text{B3})$$

Knowing the width, we can solve this equation for  $|f_{K^{*+}K^+\pi^0}|$ , while the sign of the coupling constant remains undetermined.



In the case of  $K^{*+}$  mesons, PDG [4] provides the sum of the widths of the  $K^{*+} \rightarrow K^+\pi^0$  and  $K^{*+} \rightarrow K^0\pi^+$  decays. In order to extract  $\Gamma(K^{*+} \rightarrow K^+\pi^0)$ , one has to use the relation following from isotopic invariance

$$f_{K^{*0}K^+\pi^+} = f_{K^{*+}K^0\pi^+} = \sqrt{2}f_{K^{*+}K^+\pi^0}, \quad (\text{B4})$$

and take into account mass differences between the  $K^+$  and  $K^0$  mesons and the  $\pi^+$  and  $\pi^0$  mesons:

$$\Gamma(K^{*+} \rightarrow K^+\pi^0) = \frac{\Gamma(K^{*+} \rightarrow K\pi)}{1 + 2 \left( \frac{[m_{K^{*+}}^2 - (m_{K^0} + m_{\pi^+})^2][m_{K^{*+}}^2 - (m_{K^0} - m_{\pi^+})^2]}{[m_{K^{*+}}^2 - (m_{K^+} + m_{\pi^0})^2][m_{K^{*+}}^2 - (m_{K^+} - m_{\pi^0})^2]} \right)^{\frac{3}{2}}}. \quad (\text{B5})$$

A similar expression holds for the  $K^{*0} \rightarrow K^+\pi^-$  decay mode.

$SU(3)$  symmetry allows us to obtain the remaining  $VPP$  coupling constants. Substituting matrices of the pseudoscalar octet and the vector nonet,

$$P = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & \pi^+ & K^+ \\ \frac{\pi^+}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & K^0 & \\ \frac{\bar{K}^+}{\sqrt{2}} & \frac{\bar{K}^0}{\sqrt{2}} & -\frac{2\eta}{\sqrt{6}} \end{pmatrix}, \quad V = \begin{pmatrix} \frac{\omega+\rho^0}{\sqrt{2}} & \rho^+ & K^{*+} \\ \frac{\rho^+}{\sqrt{2}} & \frac{\omega-\rho^0}{\sqrt{2}} & K^{*0} \\ \frac{\bar{K}^{*+}}{\sqrt{2}} & \frac{\bar{K}^{*0}}{\sqrt{2}} & \phi \end{pmatrix}, \quad (\text{B6})$$

into the expression for decay amplitude, we get:

$$\begin{aligned} A_{VPP} &= f_{VPP} \text{tr}[V_\mu(Pi\partial^\mu P - i\partial^\mu PP)] \\ &= if_{VPP} \left\{ \frac{1}{\sqrt{2}} K_\mu^{*+} (\bar{K}^+ \partial^\mu \pi^0 - \pi^0 \partial^\mu \bar{K}^+) + K_\mu^{*+} (\bar{K}^0 \partial^\mu \pi^+ - \pi^+ \partial^\mu \bar{K}^0) \right. \\ &\quad + K_\mu^{*0} (\bar{K}^+ \partial^\mu \pi^+ - \pi^+ \partial^\mu \bar{K}^+) - \rho_\mu^+ (\bar{K}^+ \partial^\mu K^0 - K^0 \partial^\mu \bar{K}^+) \\ &\quad + \frac{1}{2\sqrt{2}} (\omega_\mu + \rho_\mu^0) (K^+ \partial^\mu \bar{K}^+ - \bar{K}^+ \partial^\mu K^+) - \frac{1}{2} \phi_\mu (K^+ \partial^\mu \bar{K}^+ - \bar{K}^+ \partial^\mu K^+) \Big\}, \\ &\quad + \text{h.c.}, \end{aligned} \quad (\text{B7})$$

where only the terms entering (B1) are given. Comparing this expression with the corresponding parts of Lagrangian (B1), we get:

$$\begin{aligned} f_{\rho^+K^+K^0} &= -f_{K^{*+}K^0\pi^+}, & f_{\rho^0K^+K^+} &= f_{K^{*+}K^0\pi^+}/\sqrt{2}, \\ f_{\omega K^+K^+} &= f_{\rho^0K^+K^+}, & f_{\phi K^+K^+} &= -\sqrt{2}f_{\rho^0K^+K^+}. \end{aligned} \quad (\text{B8})$$

We use for  $f_{\phi K^+K^+}$  the value which follows from the direct measurement of the corresponding width. Thus, Eqs. (B3)–(B5) and (B8) determine all of the  $f_{VPP}$  values which we need, while their common sign remains undetermined.

Next, consider the  $K^{*+} \rightarrow K^+\gamma$  decay. Its amplitude can be represented in the following way:

$$A(K^{*+} \rightarrow K^+\gamma) = f_{K^{*+}K^+\gamma} \varepsilon^{\mu\nu\rho\sigma} K_\mu^{*+} \epsilon_\nu k_\rho q_\sigma K^+, \quad (\text{B9})$$

with momenta of particles shown in Fig. 8b. The numerical value of the coupling constant is determined by the width,

$$\Gamma(K^{*+} \rightarrow K^+\gamma) = \frac{f_{K^{*+}K^+\gamma}^2 m_{K^{*+}}^3}{96\pi} \left( 1 - \frac{m_{K^+}^2}{m_{K^{*+}}^2} \right)^3. \quad (\text{B10})$$

$SU(3)$  symmetry of strong interactions allows us to obtain other  $VP\gamma$  coupling constants since the decay amplitudes are proportional to

$$f_{VP\gamma} \sim \text{tr}[(PV + VP)Q], \quad (\text{B11})$$

where  $P$  and  $V$  are defined in (B6), and  $Q$  is the matrix of the quark electric charges:

$$Q = \begin{pmatrix} 2/3 & 0 & 0 \\ 0 & -1/3 & 0 \\ 0 & 0 & -1/3 \end{pmatrix}. \quad (\text{B12})$$

Thus, we get

$$f_{\rho^+\pi^+\gamma} = f_{\rho^0\pi^0\gamma} = f_{K^{*+}K^+\gamma}, \quad f_{\omega\pi^0\gamma} = 3f_{K^{*+}K^+\gamma}, \quad f_{K^{*0}K^0\gamma} = -2f_{K^{*+}K^+\gamma}, \quad (\text{B13})$$

and relative signs of the  $VP\gamma$  coupling constants are fixed. Absolute values of these constants entering Table 1 are determined from the decay widths; the sign of  $f_{\phi\pi^0\gamma}$  remains undetermined. However, the  $\phi$ -meson contribution to the decay amplitude can be neglected; see the comment after Eq. (15).

We do not add the term proportional to  $\text{tr}[(PV - VP)Q]$  in (B11) since it will change the value of  $f_{\rho^+\pi^+\gamma}$ , while the value of  $f_{\rho^0\pi^0\gamma}$  will not be changed. In this way, the relation  $f_{\rho^+\pi^+\gamma} = f_{\rho^0\pi^0\gamma}$ —which follows from the fact that, in this decay, only the isoscalar part of photon contributes—will be violated.

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